GLOBAL EDITION

Introduction to Management Science

TWELFTH EDITION

Bernard W. Taylor III

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Introduction to Management Science

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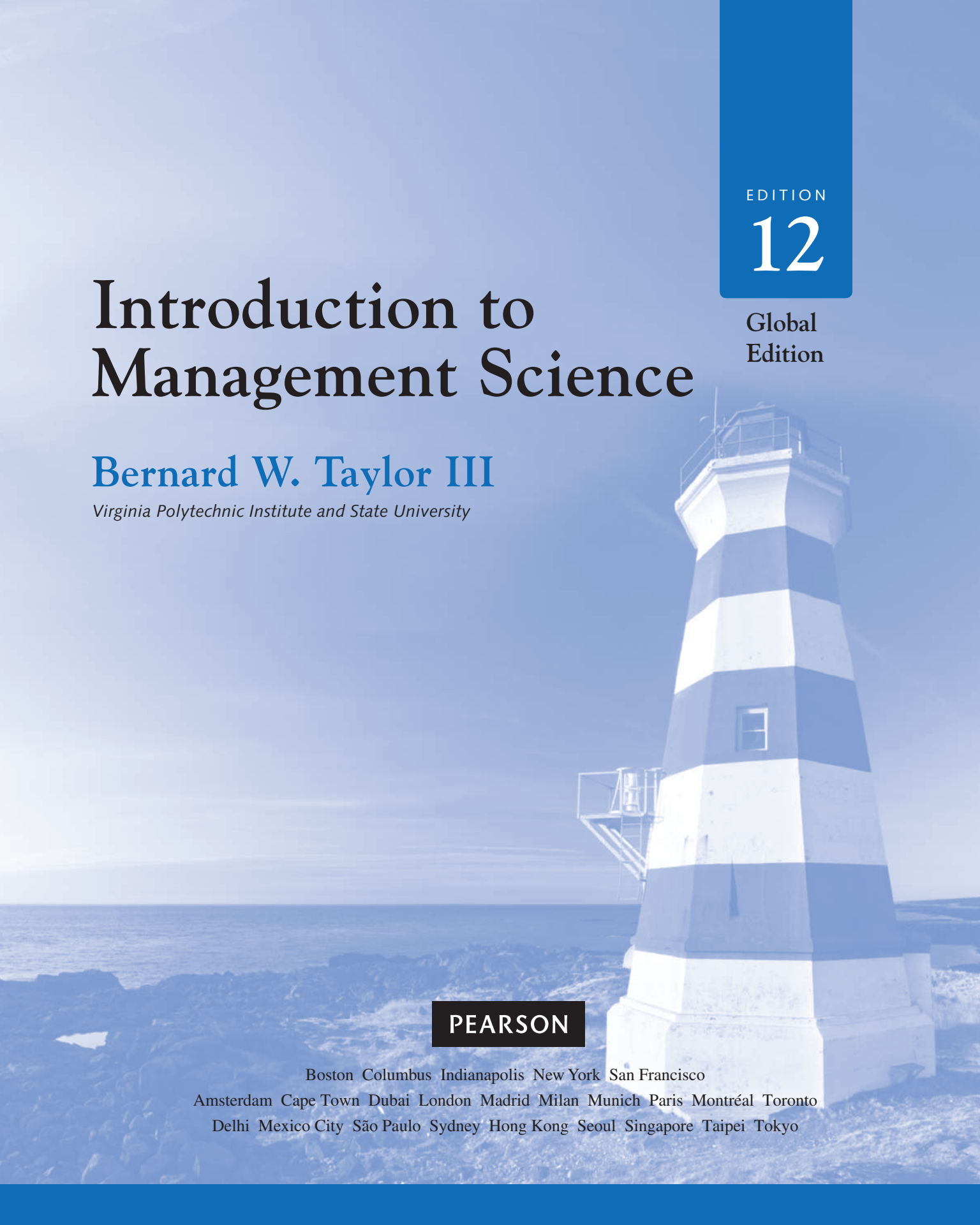
EDITION
12
Global Edition

Introduction to Management Science

Bernard W. Taylor III

Virginia Polytechnic Institute and State University



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To Diane, Kathleen, and Lindsey

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Module E: Game Theory E-1

Module F: Markov Analysis F-1

Preface

The objective of management science is to solve the decision-making problems that confront and confound managers in both the public and the private sector by developing mathematical models of those problems. These models have traditionally been solved with various mathematical techniques, all of which lend themselves to specific types of problems. Thus, management science as a field of study has always been inherently mathematical in nature, and as a result sometimes complex and rigorous. When I began writing the first edition of this book in 1979, my main goal was to make these mathematical topics seem less complex and thus more palatable to undergraduate business students. To achieve this goal I started out by trying to provide simple, straightforward explanations of often difficult mathematical topics. I tried to use lots of examples that demonstrated in detail the fundamental mathematical steps of the modeling and solution techniques. Although in the past three decades the emphasis in management science has shifted away from strictly mathematical to mostly computer solutions, my objective has not changed. I have provided clear, concise explanations of the techniques used in management science to model problems, and provided many examples of how to solve these models on the computer while still including some of the fundamental mathematics of the techniques.

The stuff of management science can seem abstract, and students sometimes have trouble perceiving the usefulness of quantitative courses in general. I remember that when I was a student, I could not foresee how I would use such mathematical topics (in addition to a lot of the other things I learned in college) in any job after graduation. Part of the problem is that the examples used in books often do not seem realistic. Unfortunately, examples must be made simple to facilitate the learning process. Larger, more complex examples reflecting actual applications would be too complex to help the student learn the modeling technique. The modeling techniques presented in this text are, in fact, used extensively in the business world, and their use is increasing rapidly because of computer and information technology, and the emerging field of business analytics. Therefore, the chances that students will use the modeling techniques that they learn from this text in a future job are very great indeed.

Even if these techniques are not used on the job, the logical approach to problem solving embodied in management science is valuable for all types of jobs in all types of organizations. Management science consists of more than just a collection of mathematical modeling techniques; it embodies a philosophy of approaching a problem in a logical manner, as does any science. Thus, this text not only teaches specific techniques but also provides a very useful method for approaching problems.

My primary objective throughout all revisions of this text is readability. The modeling techniques presented in each chapter are explained with straightforward examples that avoid lengthy written explanations. These examples are organized in a logical step-by-step fashion that the student can subsequently apply to the problems at the end of each chapter. I have tried to avoid complex mathematical notation and formulas wherever possible. These various factors will, I hope, help make the material more interesting and less intimidating to students.

New to This Edition

Management science is the application of mathematical models and computing technology to help decision makers solve problems. Therefore, new text revisions like this one tend to focus on the latest technological advances used by businesses and organizations for solving problems, as well as new features that students and instructors have indicated would be helpful to them in learning about management science. Following is a list of the substantial new changes made for this 12th edition of the text:

- This revision incorporates the latest version of Excel® 2013, and includes more than 175 new spreadsheet screenshots.
- More than 50 new exhibit screenshots have been added to show the latest versions of Microsoft® Project 2010, QM for Windows, Excel QM, TreePlan, and Crystal Ball.

- This edition includes 45 new end-of-chapter homework problems and 5 new cases, so it now contains more than 840 homework problems and 69 cases.
- All 800-plus Excel homework files on the Instructor's Web site have been replaced with new Excel 2013 files.
- Updated "Chapter Web links" are included for every chapter. More than 550 Web links are provided to access tutorials, summaries, and notes available on the Internet for the various topics in the chapters. Also included are links to YouTube videos that provide additional learning resources.
- Over 35% of the "Management Science Application" boxes are new for this edition. All of these new boxes provide current, updated applications of management science techniques by companies and organizations.
- New sections have been added on business analytics (in Chapter 1), project risk (in Chapter 8 on project management) and data mining (in Chapter 15 on forecasting).

Learning Features

This 12th edition of *Introduction to Management Science* includes many features that are designed to help sustain and accelerate a student's learning of the material. Several of the strictly mathematical topics—such as the simplex and transportation solution methods—are included as chapter modules on the Companion Web site, at **www.pearsonglobaleditions.com/Taylor**. This frees up text space for additional modeling examples in several of the chapters, allowing more emphasis on computer solutions such as Excel spreadsheets, and additional homework problems. In the following sections, we will summarize these and other learning features that appear in the text.

Text Organization

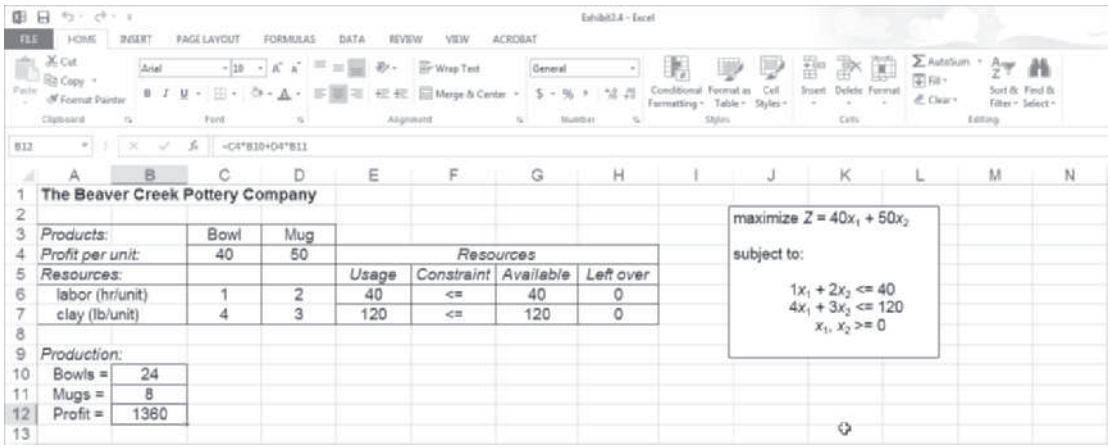
An important objective is to have a well-organized text that flows smoothly and follows a logical progression of topics, placing the different management science modeling techniques in their proper perspective. The first 10 chapters are related to mathematical programming that can be solved using Excel spreadsheets, including linear, integer, nonlinear, and goal programming, as well as network techniques.

Within these mathematical programming chapters, the traditional simplex procedure for solving linear programming problems mathematically is located in Module A on the Companion Web site, at **www.pearsonglobaleditions.com/Taylor**, that accompanies this text. It can still be covered by the student on the computer as part of linear programming, or it can be excluded, without leaving a "hole" in the presentation of this topic. The integer programming mathematical branch and bound solution method (Chapter 5) is located in Module C on the Companion Web site. In Chapter 6, on the transportation and assignment problems, the strictly mathematical solution approaches, including the northwest corner, VAM, and stepping-stone methods, are located in Module B on the Companion Web site. Because transportation and assignment problems are specific types of network problems, the two chapters that cover network flow models and project networks that can be solved with linear programming, as well as traditional model-specific solution techniques and software, follow Chapter 6 on transportation and assignment problems. In addition, in Chapter 10, on nonlinear programming, the traditional mathematical solution techniques, including the substitution method and the method of Lagrange multipliers, are located in Module D on the Companion Web site.

Chapters 11 through 14 include topics generally thought of as being probabilistic, including probability and statistics, decision analysis, queuing, and simulation. Module F on Markov analysis and Module E on game theory are on the Companion Web site. Forecasting in Chapter 15 and inventory management in Chapter 16 are both unique topics related to operations management.

Excel Spreadsheets

This new edition continues to emphasize Excel spreadsheet solutions of problems. Spreadsheet solutions are demonstrated in all the chapters in the text (except for Chapter 2, on linear programming modeling and graphical solution) for virtually every management science modeling technique presented. These spreadsheet solutions are presented in optional subsections, allowing the instructor to decide whether to cover them. The text includes more than 140 new Excel spreadsheet screenshots for Excel 2013. Most of these screenshots include reference callout boxes that describe the solution steps within the spreadsheet. Files that include all the Excel spreadsheet model solutions for the examples in the text (data files) are included on the Companion Web site and can be easily downloaded by the student to determine how the spreadsheet was set up and the solution derived, and to use as templates to work homework problems. In addition, Appendix B at the end of the text provides a tutorial on how to set up and edit spreadsheets for problem solution. Following is an example of one of the Excel spreadsheet files (from Chapter 3) that is available on the Companion Web site accompanying the text.



The Beaver Creek Pottery Company

Products:	Bowl	Mug				
Profit per unit:	40	50	Resources			
Resources:			Usage	Constraint	Available	Left over
labor (hr/unit)	1	2	40	<=	40	0
clay (lb/unit)	4	3	120	<=	120	0

Production:

Bowls =	24
Mugs =	8
Profit =	1360

maximize $Z = 40x_1 + 50x_2$

subject to:

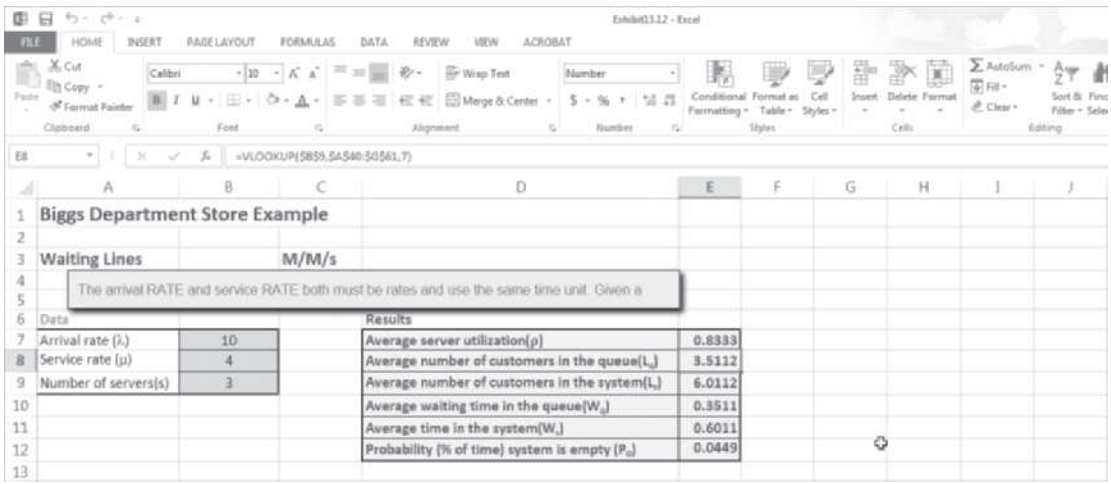
$1x_1 + 2x_2 <= 40$
$4x_1 + 3x_2 <= 120$
$x_1, x_2 >= 0$

Spreadsheet Add-Ins

Several spreadsheet add-in packages are available with this book, often in trial and premium versions. For complete information on options for downloading each package, please visit **www.pearsonglobaleditions.com/Taylor**.

Excel QM

For some management science topics, the Excel formulas that are required for solution are lengthy and complex and thus are very tedious and time-consuming to type into a spreadsheet. In several of these instances in the book, including Chapter 6 on transportation and assignment problems, Chapter 12 on decision analysis, Chapter 13 on queuing, Chapter 15 on forecasting, and Chapter 16 on inventory control, spreadsheet "add-ins" called Excel QM are demonstrated. These add-ins provide a generic spreadsheet setup with easy-to-use dialog boxes and all of the formulas already typed in for specific problem types. Unlike other "black box" software, these add-ins allow users to see the formulas used in each cell. The input, results, and the graphics are easily seen and can be easily changed, making this software ideal for classroom demonstrations and student explorations. Following below is an example of an Excel QM file (from Chapter 13 on queuing analysis) that is on the Companion Web site that accompanies the text.



Biggs Department Store Example

Waiting Lines M/M/s

The arrival RATE and service RATE both must be rates and use the same time unit. Given a

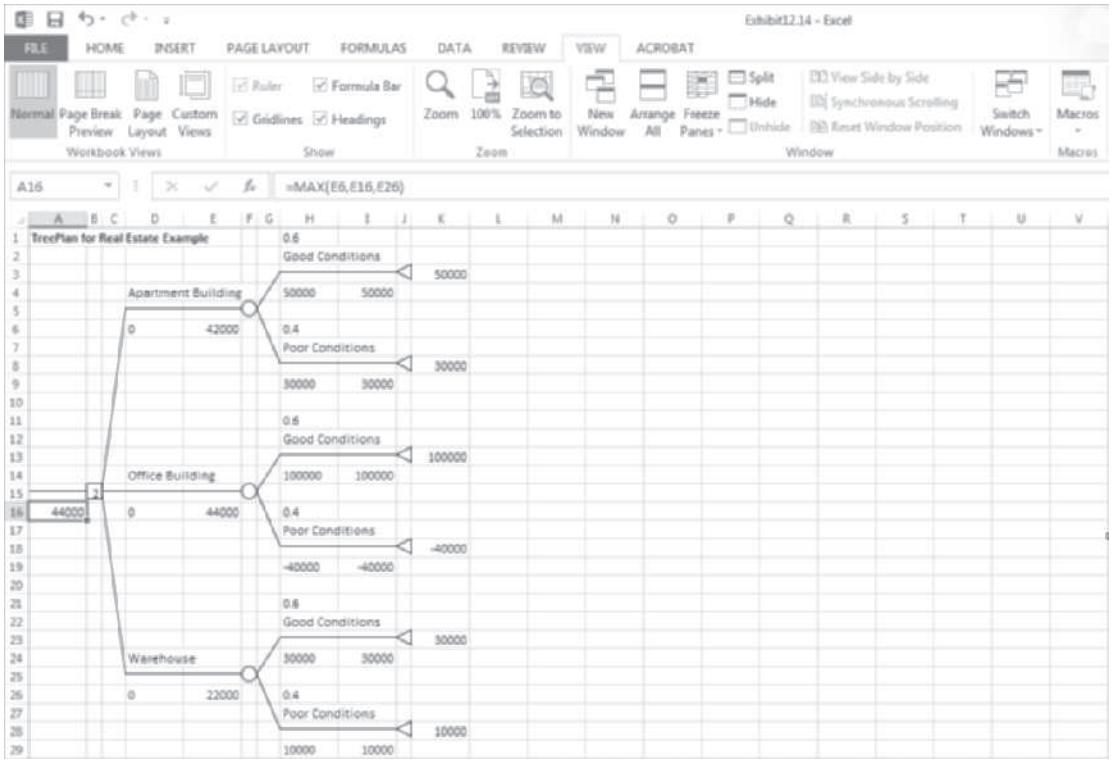
Data		Results	
Arrival rate (λ)	10	Average server utilization(ρ)	0.8333
Service rate (μ)	4	Average number of customers in the queue(L_q)	3.5112
Number of servers(s)	3	Average number of customers in the system(L_s)	6.0112
		Average waiting time in the queue(W_q)	0.3511
		Average time in the system(W_s)	0.6011
		Probability (% of time) system is empty (P_o)	0.0449

Risk Solver Platform for Education

This program is a tool for risk analysis, simulation, and optimization in Excel. The Companion Web site will direct you to a trial version of the software.

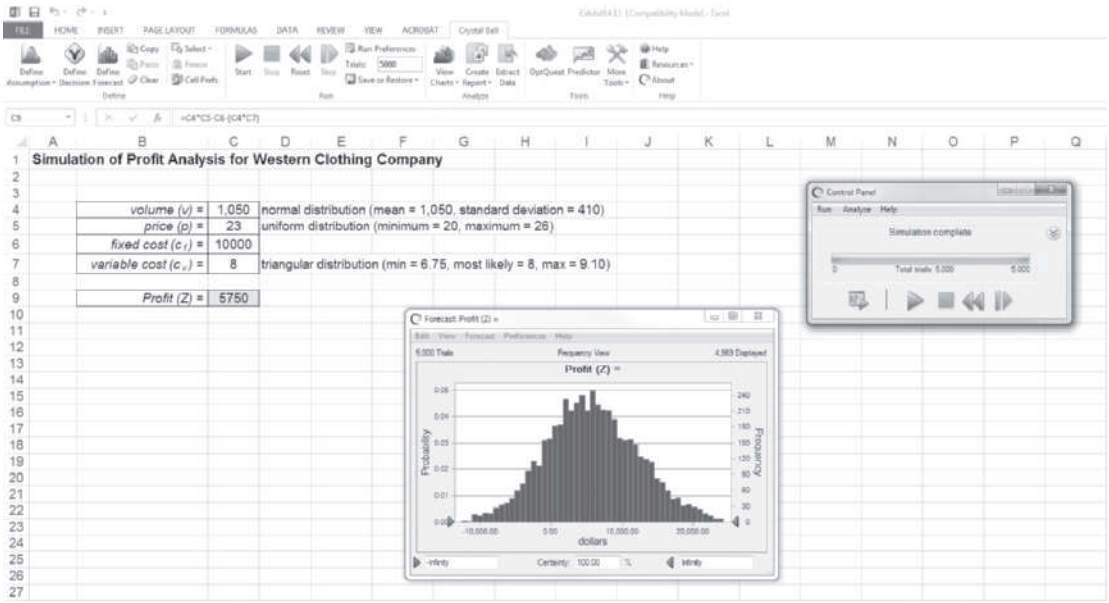
TreePlan

Another spreadsheet add-in program that is demonstrated in the text is TreePlan, a program that will set up a generic spreadsheet for the solution of decision-tree problems in Chapter 12 on decision analysis. This is also available on the Companion Web site. Following is an example of one of the **TreePlan** files (from Chapter 12) that is on the text Companion Web site.



Crystal Ball

Still another spreadsheet add-in program is Crystal Ball by Oracle. Crystal Ball is demonstrated in Chapter 14 on simulation and shows how to perform simulation analysis for certain types of risk analysis and forecasting problems. Following is an example of one of the **Crystal Ball** files (from Chapter 14) that is on the Companion Web site. The Companion Web site will direct you to a trial version of the software.



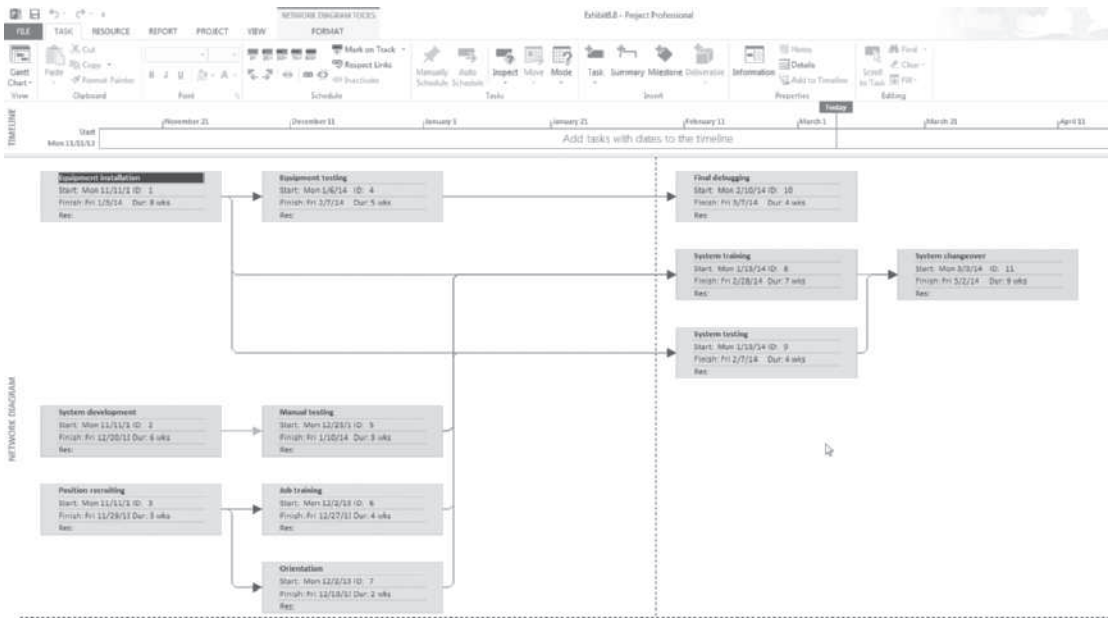
QM for Windows Software Package

QM for Windows is a computer package that is included on the text Companion Web site, and many students and instructors will prefer to use it with this text. This software is very user-friendly, requiring virtually no preliminary instruction except for the "help" screens that can be accessed directly from the program. It is demonstrated throughout the text in conjunction with virtually every management science modeling technique, except simulation. The text includes 50 QM for Windows screens used to demonstrate example problems. Thus, for most topics problem solution is demonstrated via both Excel spreadsheets and QM for Windows. Files that include all the QM for Windows solutions, for example, in the text are included on the accompanying Companion Web site. Following is an example of one of the QM for Windows files (from Chapter 4 on linear programing) that is on the Companion Web site.

Product Mix Example Solution							
	X1	X2	X3	X4		RHS	Dual
Maximize	90	125	45	65			
Processing time (hrs)	.1	.25	.08	.21	<=	72	233.3333
Shipping capacity (boxes)	3	3	1	1	<=	1,200	22.2222
Budget ($)	36	48	25	35	<=	25,000	0
Blank sweats (dozens)	1	1	0	0	<=	500	0
Blank T's (dozens)	0	0	1	1	<=	500	4.1111
Solution->	175.5556	57.7778	500	0	Optimal Z->	45,522.22	

Microsoft Project

Chapter 8 on project management includes the popular software package Microsoft Project. Following on the next page is an example of one of the Microsoft Project files (from Chapter 8) that is available on the text Companion Web site. The Companion Web site will direct you to trial version of the software.



New Problems and Cases

Previous editions of the text always provided a substantial number of homework questions, problems, and cases for students to practice on. This edition includes more than 840 homework problems, 45 of which are new, and 69 end-of-chapter case problems, 5 of which are new.

"Management Science Application" Boxes

These boxes are located in every chapter in the text. They describe how a company, an organization, or an agency uses the particular management science technique being presented and demonstrated in the chapter to compete in a global environment. There are 52 of these boxes, 18 of which are new, throughout the text. They encompass a broad range of business and public-sector applications, both foreign and domestic.

Marginal Notes

Notes in the margins of this text serve the same basic function as notes that students themselves might write in the margin. They highlight certain topics to make it easier for students to locate them, summarize topics and important points, and provide brief definitions of key terms and concepts.

Examples

The primary means of teaching the various quantitative modeling techniques presented in this text is through examples. Thus, examples are liberally inserted throughout the text, primarily to demonstrate how problems are solved with the different quantitative techniques and to make them easier to understand. These examples are organized in a logical step-by-step solution approach that the student can subsequently apply to the homework problems.

Example Problem Solutions

At the end of each chapter, just prior to the homework questions and problems, is a section that provides solved examples to serve as a guide for doing the homework problems. These examples are solved in a detailed, step-by-step fashion.

Chapter Web Links

A file on the Companion Web site contains Chapter Web links for every chapter in the text. These Web links access tutorials, summaries, and notes available on the Internet for the various techniques and topics in every chapter in the text. Also included are YouTube videos that provide additional learning resources and tutorials about many of the topics and techniques, links to the development and developers of the techniques in the text, and links to the Web sites for the companies and organizations that are featured in the "Management Science Application" boxes in every chapter. The "Chapter Web links" file includes more than 550 Web links.

Instructor Resources

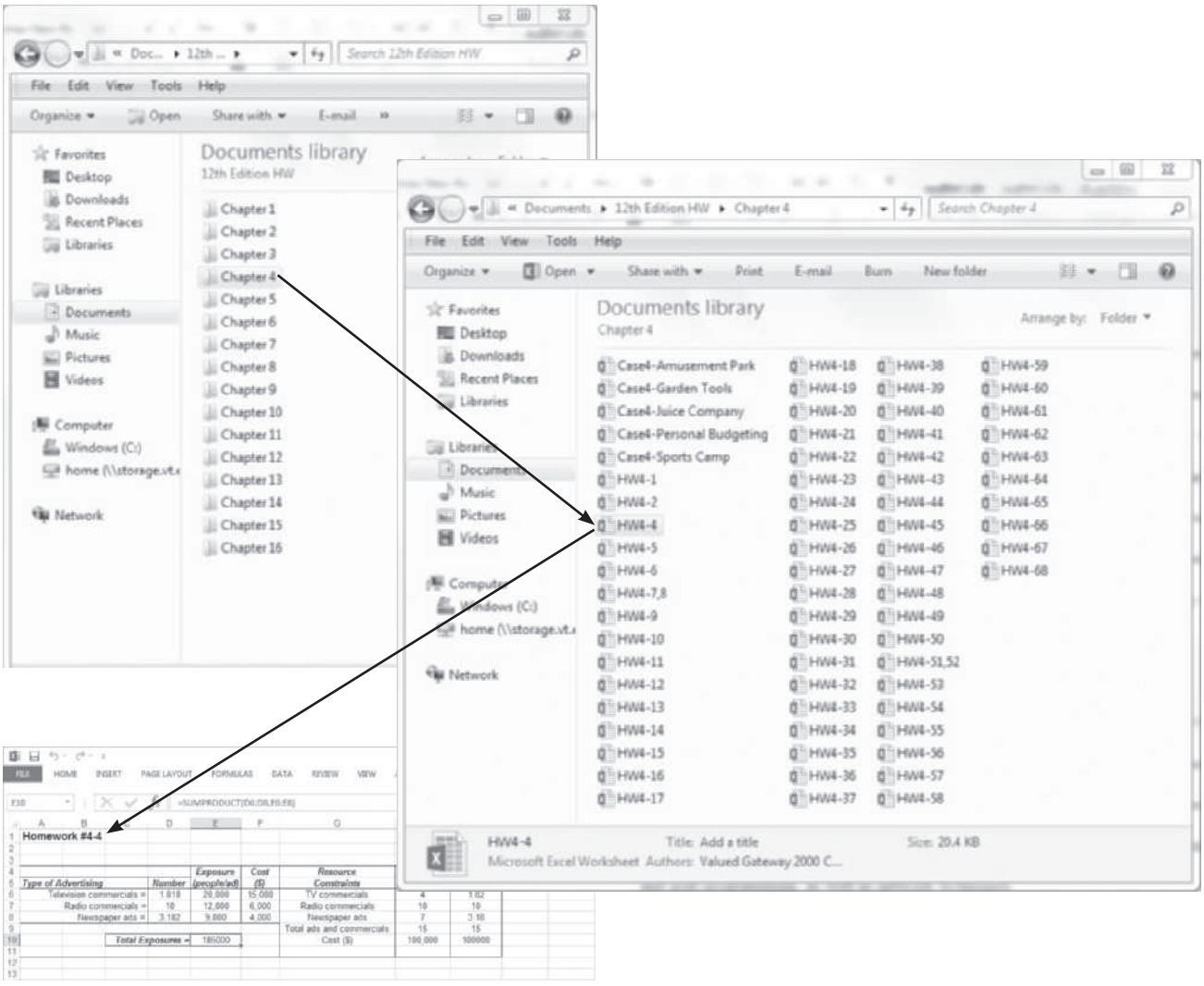
Instructor's Resource Center

At the Instructor Resource Center, **www.pearsonglobaleditions.com/Taylor**, instructors can easily register to gain access to a variety of instructor resources

available with this text in downloadable format. If assistance is needed, our dedicated technical support team is ready to help with the media supplements that accompany this text. Visit http://247.pearsoned.com for answers to frequently asked questions and toll-free user support phone numbers.

The following supplements are available with this text:

- **Instructor's Solutions Manual** The Instructor's Solutions Manual contains detailed solutions for all end-of-chapter exercises and cases. There is one file per chapter and is provided in MS Word format.
- **Excel Homework Solutions** Almost every end-of-chapter homework and case problem in this text has a corresponding Excel solution file for the instructor. This new edition includes 840 end-of-chapter homework problems, and Excel solutions are provided for all but a few of them. Excel solutions are also provided for most of the 69 end-of-chapter case problems. These Instructor Data Files are posted under the Instructor's Solutions Manual. They are organized by chapter and file type, as shown in the Chapter 4 example below. These Excel files also include those homework and case problem solutions using TreePlan (from Chapter 12) and those using Crystal Ball (from Chapter 14). In addition, Microsoft Project solution files are available for homework problems in Chapter 8.



- **Test Bank:** The Test Bank, revised by Geoff Willis of the University of Central Oklahoma College of Business, contains more than 2,000 questions, including a variety of true/false, multiple-choice, and problem-solving questions for each chapter. Each question is followed by the correct answer, the page references, the main headings, difficulty rating, and key words.
- **TestGen® Computerized Test Bank** Pearson Education's test-generating software is PC and Mac compatible and preloaded with all of the Test Bank questions. You can manually or randomly view test questions and drag and drop to create a test. You can add or modify test bank questions as needed. Conversions for use in other learning management systems are also available.
- **PowerPoint Presentations** PowerPoint presentations, revised by Geoff Willis of the University of Central Oklahoma College of Business, are available for every chapter to enhance lectures. They feature figures, tables, Excel, and main points from the text.

Student Resources

Companion Web Site

The Companion Web site for this text (**www.pearsonglobaleditions.com/Taylor**) contains the following:

- **Chapter Web Links**—provide access to tutorials, summaries, notes, and YouTube videos.
- **Data Files**—are found throughout the text; these exhibits demonstrate example problems, using Crystal Ball, Excel, Excel QM, Microsoft Project, QM for Windows, and TreePlan.
- **Online Modules**—PDF files of the online modules listed in the table of contents.
- **TreePlan**—link to download software
- **Excel QM and QM for Windows**—link to download software
- **Risk Solver Platform**—link to a free trial version
- **Crystal Ball**—link to a free trial version
- **Microsoft Project**—link to a free trial version

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Contributors

Stefania Paladini
Coventry University, UK

Xavier Pierron
Coventry University, UK

Subramaniam Ponnaiyan
American University in Dubai

Reviewers

Khaliq Ahmad Bin Mohn
International Islamic University Malaysia

Wong Wai Peng
Universiti Sains Malaysia

Ravichanran Subramaniam
Monash University Sunway Malaysia

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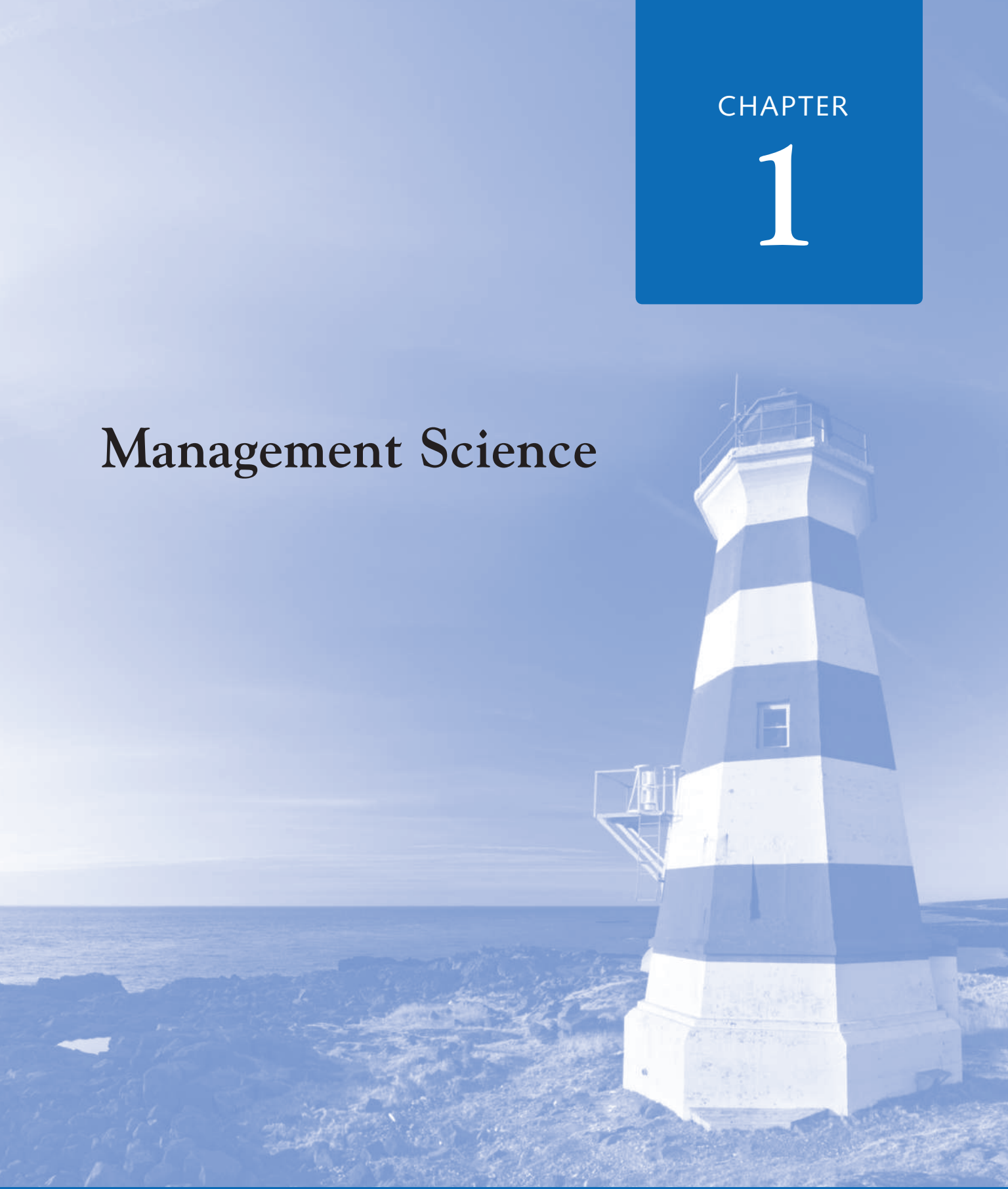
CHAPTER

1

Management Science



Management science is the application of a scientific approach to solving management problems in order to help managers make better decisions. As implied by this definition, management science encompasses a number of mathematically oriented techniques that have either been developed within the field of management science or been adapted from other disciplines, such as the natural sciences, mathematics, statistics, and engineering. This text provides an introduction to the techniques that make up management science and demonstrates their applications to management problems.

Management science is a scientific approach to solving management problems.

Management science is a recognized and established discipline in business. The applications of management science techniques are widespread, and they have been frequently credited with increasing the efficiency and productivity of business firms. In various surveys of businesses, many indicate that they use management science techniques, and most rate the results to be very good. Management science (also referred to as *operations research*, *quantitative methods*, *quantitative analysis*, *decision sciences*, and *business analytics*) is part of the fundamental curriculum of most programs in business.

Management science can be used in a variety of organizations to solve many different types of problems.

As you proceed through the various management science models and techniques contained in this text, you should remember several things. First, most of the examples presented in this text are for business organizations because businesses represent the main users of management science. However, management science techniques can be applied to solve problems in different types of organizations, including services, government, military, business and industry, and health care.

Second, in this text all of the modeling techniques and solution methods are mathematically based. In some instances the manual, mathematical solution approach is shown because it helps one understand how the modeling techniques are applied to different problems. However, a computer solution is possible for each of the modeling techniques in this text, and in many cases the computer solution is emphasized. The more detailed mathematical solution procedures for many of the modeling techniques are included as supplemental modules on the companion Web site for this text.

Management science encompasses a logical approach to problem solving.

Finally, as the various management science techniques are presented, keep in mind that management science is more than just a collection of techniques. Management science also involves the philosophy of approaching a problem in a logical manner (i.e., a scientific approach). The logical, consistent, and systematic approach to problem solving can be as useful (and valuable) as the knowledge of the mechanics of the mathematical techniques themselves. This understanding is especially important for those readers who do not always see the immediate benefit of studying mathematically oriented disciplines such as management science.

The Management Science Approach to Problem Solving

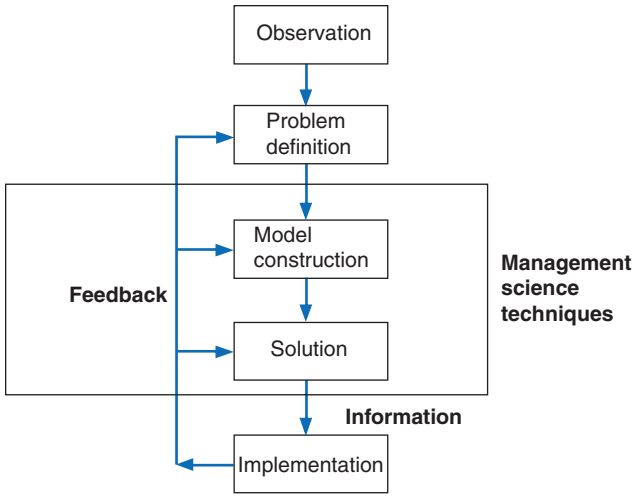
As indicated in the previous section, management science encompasses a logical, systematic approach to problem solving, which closely parallels what is known as the **scientific method** for attacking problems. This approach, as shown in Figure 1.1, follows a generally recognized and ordered series of steps: (1) observation, (2) definition of the problem, (3) model construction, (4) model solution, and (5) implementation of solution results. We will analyze each of these steps individually in this text.

*The steps of the **scientific method** are (1) observation, (2) problem definition, (3) model construction, (4) model solution, and (5) implementation.*

Observation

The first step in the management science process is the identification of a problem that exists in the system (organization). The system must be continuously and closely observed so that problems can be identified as soon as they occur or are anticipated. Problems are not always the result of a crisis that must be reacted to but, instead, frequently involve an anticipatory or planning situation. The person who normally identifies a problem is the manager because managers work in places where problems might occur. However, problems can often be identified by a

FIGURE 1.1
The management science process



*A **management scientist** is a person skilled in the application of management science techniques.*

management scientist, a person skilled in the techniques of management science and trained to identify problems, who has been hired specifically to solve problems using management science techniques.

Definition of the Problem

Once it has been determined that a problem exists, the problem must be clearly and concisely *defined*. Improperly defining a problem can easily result in no solution or an inappropriate solution. Therefore, the limits of the problem and the degree to which it pervades other units of the organization must be included in the problem definition. Because the existence of a problem implies that the objectives of the firm are not being met in some way, the goals (or objectives) of the organization must also be clearly defined. A stated objective helps to focus attention on what the problem actually is.

Model Construction

*A **model** is an abstract mathematical representation of a problem situation.*

A management science **model** is an abstract representation of an existing problem situation. It can be in the form of a graph or chart, but most frequently a management science model consists of a set of mathematical relationships. These mathematical relationships are made up of numbers and symbols.

As an example, consider a business firm that sells a product. The product costs \$5 to produce and sells for \$20. A model that computes the total profit that will accrue from the items sold is

$$Z = \$20x - 5x$$

*A **variable** is a symbol used to represent an item that can take on any value.*

In this equation, x represents the number of units of the product that are sold, and Z represents the total profit that results from the sale of the product. The *symbols* x and Z are *variables*. The term **variable** is used because no set numeric value has been specified for these items. The number of units sold, x, and the profit, Z, can be any amount (within limits); they can vary. These two variables can be further distinguished. Z is a *dependent variable* because its value is dependent on the number of units sold; x is an *independent variable* because the number of units sold is *not* dependent on anything else (in this equation).

__Parameters__ are known, constant values that are often coefficients of variables in equations.

The numbers \$20 and \$5 in the equation are referred to as **parameters**. Parameters are constant values that are generally coefficients of the variables (symbols) in an equation.

Data are pieces of information from the problem environment.

Parameters usually remain constant during the process of solving a specific problem. The parameter values are derived from **data** (i.e., pieces of information) from the problem environment. Sometimes the data are readily available and quite accurate. For example, presumably the selling price of $20 and product cost of $5 could be obtained from the firm's accounting department and would be very accurate. However, sometimes data are not as readily available to the manager or firm, and the parameters must be either estimated or based on a combination of the available data and estimates. In such cases, the model is only as accurate as the data used in constructing the model.

*A model is a **functional relationship** that includes variables, parameters, and equations.*

The equation as a whole is known as a **functional relationship** (also called *function* and *relationship*). The term is derived from the fact that profit, Z, is a *function* of the number of units sold, x, and the equation *relates* profit to units sold.

Because only one functional relationship exists in this example, it is also the *model*. In this case, the relationship is a model of the determination of profit for the firm. However, this model does not really replicate a problem. Therefore, we will expand our example to create a problem situation.

Let us assume that the product is made from steel and that the business firm has 100 pounds of steel available. If it takes 4 pounds of steel to make each unit of the product, we can develop an additional mathematical relationship to represent steel usage:

$$4x = 100 \text{ lb. of steel}$$

This equation indicates that for every unit produced, 4 of the available 100 pounds of steel will be used. Now our model consists of two relationships:

$$Z = \$20x - 5x$$
$$4x = 100$$

We say that the profit equation in this new model is an *objective function*, and the resource equation is a *constraint*. In other words, the objective of the firm is to achieve as much profit, Z, as possible, but the firm is constrained from achieving an infinite profit by the limited amount of steel available. To signify this distinction between the two relationships in this model, we will add the following notations:

$$\text{maximize } Z = \$20x - 5x$$
$$\text{subject to}$$
$$4x = 100$$

This model now represents the manager's problem of determining the number of units to produce. You will recall that we defined the number of units to be produced as x. Thus, when we determine the value of x, it represents a potential (or recommended) *decision* for the manager. Therefore, x is also known as a *decision variable*. The next step in the management science process is to solve the model to determine the value of the decision variable.

Model Solution

A management science technique usually applies to a specific model type.

Once models have been constructed in management science, they are solved using the management science techniques presented in this text. A management science solution technique usually applies to a specific type of model. Thus, the model type and solution method are both part of the management science technique. We are able to say that *a model is solved* because the model represents a problem. When we refer to model solution, we also mean problem solution.

Time Out for Pioneers in Management Science

Throughout this text, TIME OUT boxes introduce you to the individuals who developed the various techniques that are described in the chapters. This provides a historical perspective on the development of the field of management science. In this first instance, we will briefly outline the development of management science.

Although a number of the mathematical techniques that make up management science date to the turn of the twentieth century or before, the field of management science itself can trace its beginnings to military operations research (OR) groups formed during World War II in Great Britain circa 1939. These OR groups typically consisted of a team of about a dozen individuals from different fields of science, mathematics, and the military, brought together to find solutions to military-related problems. One of the most famous of these groups—called "Blackett's circus" after its leader, Nobel Laureate P. M. S. Blackett of the University of Manchester and a former naval officer—included three physiologists, two mathematical physicists, one astrophysicist, one general physicist, two mathematicians, an Army officer, and a surveyor. Blackett's group and the other OR teams made significant contributions in improving Britain's early-warning radar system (which was instrumental in their victory in the Battle of Britain), aircraft gunnery, antisubmarine warfare, civilian defense, convoy size determination, and bombing raids over Germany.

The successes achieved by the British OR groups were observed by two Americans working for the U.S. military, Dr. James B. Conant and Dr. Vannevar Bush, who recommended that OR teams be established in the U.S. branches of the military. Subsequently, both the Air Force and Navy created OR groups.

After World War II, the contributions of the OR groups were considered so valuable that the Army, Air Force, and Navy set up various agencies to continue research of military problems. Two of the more famous agencies were the Navy's Operations Evaluation Group at MIT and Project RAND, established by the Air Force to study aerial warfare. Many of the individuals who developed OR and management science techniques did so while working at one of these agencies after World War II or as a result of their work there.

As the war ended and the mathematical models and techniques that were kept secret during the war began to be released, there was a natural inclination to test their applicability to business problems. At the same time, various consulting firms were established to apply these techniques to industrial and business problems, and courses in the use of quantitative techniques for business management began to surface in American universities. In the early 1950s, the use of these quantitative techniques to solve management problems became known as management science, and it was popularized by a book of that name by Stafford Beer of Great Britain.

For the example model developed in the previous section,

$$\begin{aligned} &\text{maximize } Z = \$20x - 5x \\ &\text{subject to} \\ &\qquad 4x = 100 \end{aligned}$$

the solution technique is simple algebra. Solving the constraint equation for x, we have

$$\begin{aligned} 4x &= 100 \\ x &= 100/4 \\ x &= 25 \text{ units} \end{aligned}$$

Substituting the value of 25 for x into the profit function results in the total profit:

$$\begin{aligned} Z &= \$20x - 5x \\ &= 20(25) - 5(25) \\ &= \$375 \end{aligned}$$

A management science solution can be either a recommended decision or information that helps a manager make a decision.

Thus, if the manager decides to produce 25 units of the product and all 25 units sell, the business firm will receive \$375 in profit. Note, however, that the value of the decision variable does not constitute an actual decision; rather, it is *information* that serves as a recommendation or guideline, helping the manager make a decision.

Some management science techniques do not generate an answer or a recommended decision. Instead, they provide *descriptive results:* results that describe the system being modeled. For

example, suppose the business firm in our example desires to know the average number of units sold each month during a year. The monthly *data* (i.e., sales) for the past year are as follows:

Month	Sales	Month	Sales
January	30	July	35
February	40	August	50
March	25	September	60
April	60	October	40
May	30	November	35
June	25	December	50
		Total	480 units

Monthly sales average 40 units (480 ÷ 12). This result is not a decision; it is information that describes what is happening in the system. The results of the management science

Management Science Application

Room Pricing with Management Science at Marriott

Marriott International, Inc., headquartered in Bethesda, Maryland, has more than 140,000 employees working at more than 3,300 hotels in 70 countries. Its hotel franchises include Marriott, JW Marriott, The Ritz-Carlton, Renaissance, Residence Inn, Courtyard, TownePlace Suites, Fairfield Inn, and Springhill Suites. *Fortune* magazine ranks Marriott as the lodging industry's most admired company and one of the best companies to work for.

Marriott uses a revenue management system for individual hotel bookings. This system provides forecasts of customer demand and pricing controls, makes optimal inventory allocations, and interfaces with a reservation system that handles more than 75 million transactions each year. The system makes a demand forecast for each rate category and length of stay for each arrival day up to 90 days in advance, and it provides inventory allocations to the reservation system. This inventory of hotel rooms is then sold to individual customers through channels such as Marriott.com, the company's toll-free reservation number, the hotels directly, and global distribution systems.

One of the most significant revenue streams for Marriott is for group sales, which can contribute more than half of a full-service hotel's revenue. However, group business has challenging characteristics that introduce uncertainty and make modeling it difficult, including longer booking windows (as compared to those for individuals), price negotiation as part of the booking process, demand for blocks of rooms, and lack of demand data. For a group request, a hotel must know if it has sufficient rooms and determine a recommended rate.

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A key challenge is estimating the value of the business the hotel is turning away if the room inventory is given to a group rather than being held for individual bookings.

To address the group booking process, Marriott developed a decision support system, Group Pricing Optimizer (GPO), that provides guidance to Marriott personnel on pricing hotel rooms for group customers. GPO uses various management science modeling techniques and tools, including simulation, forecasting, and optimization techniques, to recommend an optimal price rate. Marriott estimates that GPO provided an improvement in profit of over $120 million derived from $1.3 billion in group business in its first 2 years of use.

Source: Based on S. Hormby, J. Morrison, P. Dave, M. Myers, and T. Tenca, "Marriott International Increases Revenue by Implementing a Group Pricing Optimizer," *Interfaces* 40, no. 1 (January–February 2010): 47–57.

techniques in this text are examples of the two types shown in this section: (1) solutions/decisions and (2) descriptive results.

Implementation

Implementation is the actual use of a model once it has been developed.

The final step in the management science process for problem solving described in Figure 1.1 is implementation. **Implementation** is the actual use of the model once it has been developed or the solution to the problem the model was developed to solve. This is a critical but often overlooked step in the process. It is not always a given that once a model is developed or a solution found, it is automatically used. Frequently the person responsible for putting the model or solution to use is not the same person who developed the model, and thus the user may not fully understand how the model works or exactly what it is supposed to do. Individuals are also sometimes hesitant to change the normal way they do things or to try new things. In this situation, the model and solution may get pushed to the side or ignored altogether if they are not carefully explained and their benefit fully demonstrated. If the management science model and solution are not implemented, then the effort and resources used in their development have been wasted.

Management Science and Business Analytics

Analytics is the latest hot topic and new buzzword in business. Companies are establishing analytics departments and the demand for employees with analytics skills and expertise is growing faster than almost any other business skill set. Universities and business schools are developing new degree programs and courses in analytics. So exactly what is this new and very popular area called business analytics and how does it relate to management science?

Business analytics uses large amounts of data with management science techniques and modeling to help managers makes decisions.

Business analytics is a somewhat general term that seems to have a number of different definitions, but in broad terms it is considered to be a process for using large amounts of data combined with information technology, statistics, management science techniques, and mathematical modeling to help managers solve problems and make decisions that will improve their business performance. It makes use of these technological tools to help businesses understand their past performance and to help them plan and make decisions for the future; thus analytics is said to be descriptive, predictive, and prescriptive.

A key component of business analytics is the recent availability of large amounts of data—called "big data"—that is now accessible to businesses, and that is perceived to be an integral part and starting point of the analytical process. Data are considered to be the engine that drives the process of analysis and decision making in business analytics. For example, a bank might apply analytics by using data to determine different customer characteristics in order to match them with the bank services they provide; or a retail store might apply analytics by using data to determine which styles of denim jeans match their customer preferences, determine how many jeans to order from their foreign suppliers, how much inventory to keep on hand, and when the best time is to sell the jeans and what is the best price.

If you have not already noticed, analytics is very much like the "management science approach to problem solving" that we have already described in the previous section. In fact, many in business perceive business analytics to just be a repackaged version of management science. In some business schools, management science courses are simply being renamed as "analytics." Business students are being advised that in the future companies will expect them to have an analytics skill set and these skills need to include knowledge of statistics, mathematical modeling, and quantitative tools—the topics traditionally considered to be management science and that are covered in this text.

For our purposes in studying management science, it is clear that the quantitative tools and techniques that are included in this book are an important major part of business analytics, no matter what the definition of the business analytics process is. As such, becoming skilled in the use of these management science techniques is a necessary and important step for someone who wants to become a business analytics professional.

Model Building: Break-Even Analysis

Break-even analysis is a modeling technique to determine the number of units to sell or produce that will result in zero profit.

In the previous section, we gave a brief, general description of how management science models are formulated and solved, using a simple algebraic example. In this section, we will continue to explore the process of building and solving management science models, using **break-even analysis**, also called *profit analysis*. Break-even analysis is a good topic to expand our discussion of model building and solution because it is straightforward, relatively familiar to most people, and not overly complex. In addition, it provides a convenient means to demonstrate the different ways management science models can be solved—mathematically (by hand), graphically, and with a computer.

The purpose of break-even analysis is to determine the number of units of a product (i.e., the volume) to sell or produce that will equate total revenue with total cost. The point where total revenue equals total cost is called the *break-even point*, and at this point profit is zero. The break-even point gives a manager a point of reference in determining how many units will be needed to ensure a profit.

Components of Break-Even Analysis

The three components of break-even analysis are volume, cost, and profit. *Volume* is the level of sales or production by a company. It can be expressed as the number of units (i.e., quantity) produced and sold, as the dollar volume of sales, or as a percentage of total capacity available.

Fixed costs are independent of volume and remain constant.

Two types of cost are typically incurred in the production of a product: fixed costs and variable costs. **Fixed costs** are generally independent of the volume of units produced and sold. That is, fixed costs remain constant, regardless of how many units of product are produced within a given range. Fixed costs can include such items as rent on plant and equipment, taxes, staff and management salaries, insurance, advertising, depreciation, heat and light, and plant maintenance. Taken together, these items result in total fixed costs.

Variable costs depend on the number of items produced.

Variable costs are determined on a per-unit basis. Thus, total variable costs depend on the number of units produced. Variable costs include such items as raw materials and resources, direct labor, packaging, material handling, and freight.

Total variable costs are a function of the *volume* and the *variable cost per unit*. This relationship can be expressed mathematically as

$$\text{total variable cost} = vc_v$$

where c_v = variable cost per unit and v = volume (number of units) sold.

Total cost (TC) *equals the fixed cost* (c_f) *plus the variable cost per unit* (c_v) *multiplied by volume* (v).

The **total cost** of an operation is computed by summing total fixed cost and total variable cost, as follows:

$$\text{total cost} = \text{total fixed cost} + \text{total variable cost}$$

or

$$TC = c_f + vc_v$$

where c_f = fixed cost.

As an example, consider Western Clothing Company, which produces denim jeans. The company incurs the following monthly costs to produce denim jeans:

$$\text{fixed cost} = c_f = \$10{,}000$$
$$\text{variable cost} = c_v = \$8 \text{ per pair}$$

If we arbitrarily let the monthly sales volume, v, equal 400 pairs of denim jeans, the total cost is

$$TC = c_f + vc_v = \$10{,}000 + (400)(8) = \$13{,}200$$

Profit is the difference between total revenue (volume multiplied by price) and total cost.

The third component in our break-even model is **profit**. Profit is the difference between *total revenue* and total cost. Total revenue is the volume multiplied by the price per unit,

$$\text{total revenue} = vp$$

where p = price per unit.

For our clothing company example, if denim jeans sell for \$23 per pair and we sell 400 pairs per month, then the total monthly revenue is

$$\text{total revenue} = vp = (400)(23) = \$9{,}200$$

Now that we have developed relationships for total revenue and total cost, profit (Z) can be computed as follows:

$$\begin{aligned}\text{total profit} &= \text{total revenue} - \text{total cost}\\ Z &= vp - (c_f + vc_v)\\ &= vp - c_f - vc_v\end{aligned}$$

Computing the Break-Even Point

For our clothing company example, we have determined total revenue and total cost to be \$9,200 and \$13,200, respectively. With these values, there is no profit but, instead, a loss of \$4,000:

$$\text{total profit} = \text{total revenue} - \text{total cost} = \$9{,}200 - 13{,}200 = -\$4{,}000$$

We can verify this result by using our total profit formula,

$$Z = vp - c_f - vc_v$$

and the values $v = 400$, $p = \$23$, $c_f = \$10{,}000$, and $c_v = \$8$:

$$\begin{aligned}Z &= vp - c_f - vc_v\\ &= \$(400)(23) - 10{,}000 - (400)(8)\\ &= \$9{,}200 - 10{,}000 - 3{,}200\\ &= -\$4{,}000\end{aligned}$$

Obviously, the clothing company does not want to operate with a monthly loss of \$4,000 because doing so might eventually result in bankruptcy. If we assume that price is static because of market conditions and that fixed costs and the variable cost per unit are not subject to change, then the only part of our model that can be varied is *volume*. Using the modeling terms we developed earlier in this chapter, price, fixed costs, and variable costs are parameters, whereas the volume, v, is a decision variable. In break-even analysis, we want to compute the value of v that will result in zero profit.

*The **break-even point** is the volume* (v) *that equates total revenue with total cost where profit is zero.*

At the **break-even point**, where total revenue equals total cost, the profit, Z, equals zero. Thus, if we let profit, Z, equal zero in our total profit equation and solve for v, we can determine the break-even volume:

$$\begin{aligned}Z &= vp - c_f - vc_v\\ 0 &= v(23) - 10{,}000 - v(8)\\ 0 &= 23v - 10{,}000 - 8v\\ 15v &= 10{,}000\\ v &= 666.7 \text{ pairs of jeans}\end{aligned}$$

In other words, if the company produces and sells 666.7 pairs of jeans, the profit (and loss) will be zero and the company will *break even*. This gives the company a point of reference from which to determine how many pairs of jeans it needs to produce and sell in order to gain a profit

(subject to any capacity limitations). For example, a sales volume of 800 pairs of denim jeans will result in the following monthly profit:

$$\begin{aligned} Z &= vp - c_f - vc_v \\ &= \$(800)(23) - 10{,}000 - (800)(8) = \$2{,}000 \end{aligned}$$

In general, the break-even volume can be determined using the following formula:

$$\begin{aligned} Z &= vp - c_f - vc_v \\ 0 &= v(p - c_v) - c_f \\ v(p - c_v) &= c_f \\ v &= \frac{c_f}{p - c_v} \end{aligned}$$

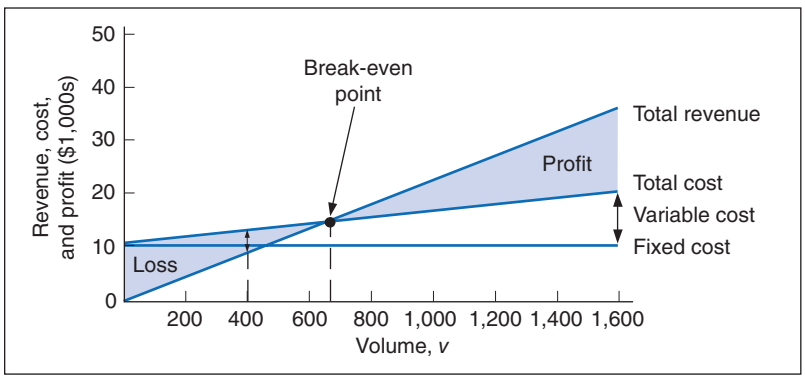
For our example,

$$\begin{aligned} v &= \frac{c_f}{p - c_v} \\ &= \frac{10{,}000}{23 - 8} \\ &= 666.7 \text{ pairs of jeans} \end{aligned}$$

Graphical Solution

It is possible to represent many of the management science models in this text graphically and use these graphical models to solve problems. Graphical models also have the advantage of providing a "picture" of the model that can sometimes help us understand the modeling process better than mathematics alone can. We can easily graph the break-even model for our Western Clothing Company example because the functions for total cost and total revenue are *linear*. That means we can graph each relationship as a straight line on a set of coordinates, as shown in Figure 1.2.

FIGURE 1.2
Break-even model



In Figure 1.2, the fixed cost, c_f, has a constant value of \$10,000, regardless of the volume. The total cost line, *TC*, represents the sum of variable cost and fixed cost. The total cost line increases because variable cost increases as the volume increases. The total revenue line also increases as volume increases, but at a faster rate than total cost. The point where these two lines intersect indicates that total revenue equals total cost. The volume, v, that corresponds to this point is the *break-even volume*. The break-even volume in Figure 1.2 is 666.7 pairs of denim jeans.

Sensitivity Analysis

We have now developed a general relationship for determining the break-even volume, which was the objective of our modeling process. This relationship enables us to see how the level of profit (and loss) is directly affected by changes in volume. However, when we developed this